Non split two out degree equitable domination number in graphs

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ABSTRACT

Let G=(V,E) be a undirected graph. A dominating D of G is said to two out degree equitable dominating set if for any two vertices u,vεD such that |od_p(u)−od_p(v)| ≤ 2 where od_p(u) = |N(v)∩V−D|. The minimum cardinality of two out degree equitable dominating set is called two out degree equitable domination number and it is denoted by γ_{2 oe}(G). In this paper non split two out degree equitable domination in graph is introduced and studied.

AMS Classification: 05C9

KEY WORDS: two-out degree, equitable, non-split, dominating set.

1. INTRODUCTION

The graphs considered here are nontrivial, connected, simple finite and undirected. For a graph G=(V,E), V denoted it vertex set and E its edge set. The number of vertices and edges are denoted by p and q respectively. The open neighborhood of v is denoted by N(u) and defined as N(u)= {v/uv E}. The out degree of v with respect to D is denoted by od_p(u) = |N(v)∩V−D|

The concept of domination was first studied by Ore (Kulli and Jankiram, 2000) and Berge (1962). A set D ⊆ V is said to be a dominating set of G if every vertex in V−D is adjacent to some vertex in D. The cardinality of a minimum dominating set of G is called the domination number of G and is denoted by γ(G). Ali Sahal and V.Mathad (Sahal, 2013) introduce the concept of two out degree equitable domination in graphs. A dominating D of G is said to two out degree equitable dominating set if for any two vertices u,vεD such that |od_p(u)−od_p(v)| ≤ 2. The minimum cardinality of two out degree equitable dominating set is called two out degree equitable domination number and it is denoted by γ_{2 oe}(G). M.S.Mahesh and P.Namasivayam (Mahesh, 2014) introduced the concept of connected two out degree equitable domination in graphs. A two out degree equitable dominating set is called connected two out degree equitable dominating set if the induced sub graph <D> is connected. The minimum cardinality of connected two out degree equitable dominating set is called connected two out degree equitable domination number and it is denoted by γ_{c2 oe}(G). V.Kulli and Jankiram (2000) introduced the concept of non-split domination in graphs. A dominating set of V(G) is a non-split dominating set if the induced sub graph <V−D> is connected. The minimum cardinality of non-split two out degree equitable dominating set is called non split two out degree equitable domination number and it is denoted by γ_{nss2 oe}(G).

The purpose of the paper to introduce the concept of non-split two out degree equitable domination in graphs.

Non-Split two out degree equitable domination in graphs:

Definition: A two out degree equitable dominating set D of a graph is non-split two out degree equitable dominating set if the induced sub graph <V−D> is connected. The minimum cardinality of non-split two out degree equitable dominating set is called non-split two out degree equitable domination number and it is denoted by γ_{nss2 oe}(G).

Example:

![Figure 1](image)

Let us consider a set D = {1, 3, 6, 7} and V−D= {2, 4, 5}

od_p(1) = |N(1) ∩ {2,4,5}| = 1
od_p(3) = |N(3) ∩ {2,4,5}| = 2
od_p(6) = |N(6) ∩ {2,4,5}| = 1
od_p(7) = |N(7) ∩ {2,4,5}| = 1

Then Clearly for any u,vε {1,3,6,7} such that |od_p(u)−od_p(v)| ≤ 2.

D= {1, 3, 6, 7} is two out degree equitable dominating set

Clearly <2,4,5> is connected

D= {1,3,6,7} is minimum non-split two out degree equitable dominating set

γ_{nss2 oe}(G)=4.

Observation: For any graph G with p vertices, 2≤ γ_{nss2 oe}(G) ≤ p − 2

Non-Split two out degree equitable domination number for different graphs:

October - December 2016 2266  JCPS Volume 9 Issue 4
Theorem: For any complete graph $y_{ns2oe}(k_p) = 2$

Proof:
Let $V = \{u_1, u_2, \ldots, u_p\}$ be the vertices set of $k_p$
Let $D = \{u_1, u_2\}$ be a dominating set of $G$ and $V - D = \{u_3, u_4, \ldots, u_p\}$
Now, $u_1 \in D$ then $od_D(u_1) = |N(u_1) \cap V - D|$
$= |\{u_2, u_3, u_4, \ldots, u_p\} \cap \{u_3, u_4, \ldots, u_p\}|$
$= |\{u_3, u_4, \ldots, u_p\}| = p - 2$
Similarly $od_D(u_2) = p - 2$
Then $|od_D(u_1) - od_D(u_2)| \leq 2$. For any $u_i, u_j \in D$
So $D$ is two out degree equitable dominating set
The induced sub graph $< V - D >$ is connected
$y_{ns2oe}(k_p) \leq 2$ and $2 \leq y_{ns2oe}(k_p)$
Then $y_{ns2oe}(k_p) = 2$

Theorem: If $G$ is a star $k_{1,p}$, then $y_{ns2oe}(k_{1,p}) = p - 2$

Proof:
Let $V = \{v, u_1, u_2, \ldots, u_p\}$ be the vertices set of $k_{1,p}$
Let $D = \{u_1, u_2, \ldots, u_p\}$ be a domination set of $G$ and $V - D = \{v, u_p\}$
Now, $u_i \in D$ then $od_D(u_i) = |N(u_i) \cap V - D|$
$= |\{v\} \cap \{v, u_p\}|$
$= |\{v\}| = 1$
Then $|od_D(v) - od_D(u_i)| \leq 2$. For any $u_i \in D$
So $D$ is two out degree equitable dominating set and $< V - D = \{v, u_p\} >$ is connected
D be minimal non-split two out degree equitable dominating set
then $y_{ns2oe}(k_{1,p}) = p - 2$

Theorem: For any complete bipartite graph $k_{s,t}$, is $y_{ns2oe}(k_{s,t}) = \begin{cases} 2 & \text{if } |s - t| \leq 2 \\ s + t - 2 & \text{otherwise} \end{cases}$

Proof:
Let $V = \{u_1, u_2, u_3, \ldots, u_s, v_1, v_2, v_3, \ldots, v_t\}$ be the vertices set of $k_{m,n}$ and $\{u_4, u_2, u_3, \ldots, u_s\}$
and $\{v_1, v_2, v_3, \ldots, v_t\}$ be the partition of $V$.
Case (i) $|s - t| \leq 2$
Let $D = \{u_1, v_1\}$ be a dominating set of $G$ and
$V - D = \{u_1, u_2, u_3, \ldots, u_s, v_1, v_2, v_3, \ldots, v_{t-1}, v_{t+1}, \ldots, v_t\}$
Now, $u_i \in D$ then $od_D(u_i) = |N(u_i) \cap V - D|$
$= |\{v_1, v_2, v_3, \ldots, v_{t-1}, v_{t+1}, \ldots, v_t\} - \{u_1, u_2, u_3, \ldots, u_{i-1}, u_{i+1}, \ldots, u_s, v_1, v_2, v_3, \ldots, v_{t-1}, v_{t+1}, \ldots, v_t\}|$
$= |\{v_1, v_2, v_3, \ldots, v_{t-1}, v_{t+1}, \ldots, v_t\} - | t - 1 |
\text{ if } v_j \in D \text{ then } od_D(v_j) = |N(v_j) \cap V - D|$
$= |\{u_1, u_2, u_3, \ldots, u_{i-1}, u_{i+1}, \ldots, u_s\} - \{u_1, u_2, u_3, \ldots, u_{i-1}, u_{i+1}, \ldots, u_s, v_1, v_2, v_3, \ldots, v_{t-1}, v_{t+1}, \ldots, v_t\}|$
$= |\{u_1, u_2, u_3, \ldots, u_{i-1}, u_{i+1}, \ldots, u_s\}| = s - 1$
$|od_D(u_i) - od_D(v_j)| = |t - 1 - s + 1 = t - s| \leq 2$
Then $|od_D(u_i) - od_D(v_j)| \leq 2$. For any $u_i, v_j \in D$
So $D$ is two out degree equitable dominating set and the induced sub graph $< V - D >$ is connected
$y_{ns2oe}(k_{s,t}) \leq 2$ and $2 \leq y_{ns2oe}(k_{s,t})$
$y_{ns2oe}(k_{s,t}) = 2$
Case (ii) $|s - t| \geq 2$ and $s \geq t$
Let $D = \{u_1, u_2, u_3, \ldots, u_{s-1}, v_1, v_2, v_3, \ldots, v_{t-1}\}$ be the vertices set of $k_{s,t}$ and $V - D = \{u_s, u_t\}$
Clearly $< V - D >$ is non-split two out degree equitable dominating set
Then $y_{ns2oe}(k_{s,t}) \leq s + t - 2$. and $s + t - 2 \leq y_{ns2oe}(k_{s,t})$
Then $y_{ns2oe}(k_{s,t}) \leq s + t - 2$

Theorem: For any cycle $C_p$, then $y_{ns2oe}(C_p) = p - 2$

Proof:
Let \( V=\{u_1, u_2, \ldots, u_p\} \) be the vertices set of \( C_p \)

Let \( D=\{u_1, u_2, \ldots, u_{i-1}, u_{i+2}, \ldots, u_p\} \) dominating set of \( C_p \) and \( V- D = \{u_i, u_{i+1}\} \)

Now \( od_D(u_j) = 0, j=1, 2, \ldots, i-2, i+3, \ldots, p \)

\( od_D(u_i) = 1 \) and \( od_D(u_{i+1}) = 1 \)

Then \( |od_D(u_i) - od_D(u_{i+1})| \leq 2 \)

Then \( D \) is two out degree equitable dominating set

So \( y_{2oe}(C_p) = p - 2 \), and \( <V - D> \) is connected

Then \( D \) is minimum non split two out degree equitable dominating set

\( y_{ns2oe}(C_p) = p - 2 \)

**Theorem:** For the Path \( P_p \), \( y_{ns2oe}(P_p) = p - 2 \), \( p \geq 2 \)

**Proof:**

Since the degree of any vertex in \( P_p \) is 2 except the initial and terminal vertices.

Let \( D=\{v_1, v_2, v_3, \ldots, v_{p-2}\} \) and \( V - D = \{v_{p-1}, v_p\} \)

Clearly \( D \) is two out degree equitable dominating set

Then \( <V - D> \) is connected

So \( D \) is non-split two out degree equitable dominating set

Then \( y_{ns2oe}(P_p) = p - 2 \)

**Theorem:** For the Wheel \( W_p \), \( y_{ns2oe}(W_p) = \begin{cases} 2 & \text{if } p = 4, 5 \\ p - 4 & \text{if } p \geq 7 \end{cases} \)

**Proof:**

Let \( W_p \) be a when with \( p - 1 \) vertices on the cycle and a single vertex at the center. Let \( V(W_p) = \{v, v_1, v_2, v_3, \ldots, v_{p-1}\} \)

where \( u \) is the center and \( v_i \) \((1 \leq i \leq p - 1)\) is on the cycle. Clearly \( \text{deg}(v_i) = 3 \) for all \( 1 \leq i \leq p - 1 \) and \( \text{deg}(u) = p - 1 \).

Clearly \( p \geq 4 \). We have the following cases

**Case 1. p=4 and 5**

If \( p=4 \) then \( W_4 \) forms a complete graph then by theorem 3.1 \( y_{ns2oe}(W_4) = 2 \)

If \( p=5 \). Let us take \( D=\{u, v_1\} \) and \( V - D = \{v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots, v_{p-1}\} \) since \( u \) is adjacent with \( v_i \) for all \( i \leq i \leq 4 \), \( V - D \subset N(u) \) so \( N(u) \cap V - D \subset V - D \)

\( od_D(u) = |N(u) \cap V - D| = |V - D| = 2 \)

Now for \( v_i \). Since \( \text{deg}(v_i) = 3 \) and \( v_i \) is adjacent to \( u \in D \) then \( N(v_i) = \{u, v_j, v_k\} \)

and \( N(v_i) \cap V - D = \{v_j, v_k\} \)

\( od_D(v_i) = |N(u) \cap V - D| = 2 \)

\( |od_D(u) - od_D(v_i)| = 1 \leq 2 \) and clearly \( <V - D> \) is connected

Hence \( y_{ns2oe}(W_p) \leq 2 \) and \( 2 \leq y_{ns2oe}(G) \)

Hence \( y_{ns2oe}(W_p) = 2 \)

**Case 2. p\geq6**

In this case \( \text{deg}(u) = p \), while \( \text{deg}(v_i) = 3 \) for all \( i, 1 \leq i \leq 5 \)

Let us take \( D=\{u, v_1, v_2, v_3, \ldots, v_m, v_3, \ldots, v_{m-1}\} \) be a dominating set and \( V - D = \{v_{p-3}, v_{p-2}, v_{p-1}, v_p\} \) since \( u \) is adjacent with \( v_i \) for all \( i, V - D \subset N(u) \)

so \( N(u) \cap V - D \subset V - D \)

\( od_D(u) = |N(u) \cap V - D| = 4 \)

Now for \( v_i \) and \( v_j \)

If \( v_i \) and \( v_j \) are adjacent \( N(v_i) = \{u, v_j, v_k\} \) and \( N(u) \cap V - D = \{v_k\} \)

\( od_D(v_i) = |N(u) \cap V - D| = 1 \)

If \( v_i \) and \( v_j \) are not adjacent but \( v_i \) and \( v_j \) are adjacent with \( u \) so \( N(u) \cap V - D \) contains two elements so \( od_D(v_i) = |N(u) \cap V - D| = 2 \)

So for any elements \( u, v \in D \)

\( |od_D(u) - od_D(v)| \leq 2 \) and clearly \( <V - D> = \) is connected

So \( D \) is non-split two out degree equitable dominating set

Hence \( y_{ns2oe}(W_p) = p - 4 \)

**Theorem:** For the double star \( S_{r,t} \), \( y_{ns2oe}(S_{r,t}) = r + t \)

**Proof:**

Let \( \{u_1, u_2, u_3, v_1, v_2, v_3\} \) be the vertices of \( S_{r,t} \) and all \( u_i \) is adjacent to \( u \) and \( v \) is adjacent to \( v \).
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Here \{u_1, u_2, u_3, \ldots, u_r, v_1, v_2, v_3, \ldots, v_t\} be the isolated vertices and D=\{u,v\} is support vertices and is connected
So let us take D=\{u_1, u_2, u_3, \ldots, u_r, v_1, v_2, v_3, \ldots, v_t\} and V - D =\{u,v\}
Since every vertices has one neighborhood so clearly D is two out degree equitable dominating set
Clearly \(< V - D >\) is connected
\(\gamma_{ns2oe}(S_r,t) = r + t\)

**Theorem:** For any Hoffman tree \(\gamma_{ns2oe}(P_n^+) = p\)

**Proof:**
Let \(V(P_n^+) = \{v_1, v_2, v_3, \ldots, v_p, v_{p+1}, v_{p+2}, v_{p+3}, \ldots, v_{pp}\}\)
Here \(\{v_1, v_2, v_3, \ldots, v_p\}\) be the vertices of the path, \(\{v_{p+1}, v_{p+2}, v_{p+3}, \ldots, v_{pp}\}\) be the pendant edge attached at each vertex of the path.
Let D= \(\{v_{p+1}, v_{p+2}, v_{p+3}, \ldots, v_{pp}\}\) be minimal dominating set and \(V - D = \{v_1, v_2, v_3, \ldots, v_p\}\)
Each vertices of path \(v_{pi}\) have a neighborhood in D and other \(V - D\)
\(od_D(v_{pi}) = |N(v_{pi})\cap V - D| = 1\) for all \(i=1,2,3,\ldots,n\)
\(|od_D(v_{pi}) - od_D(v_{pj})| \leq 2\)
Then D is two out degree equitable dominating set and \(<N(D)> = V - D\) form a path , so
\(< V - D >\)connected
Then D is minimal non split two out degree equitable dominating set
Then \(\gamma_{ns2oe}(P_n^+) = |D| = p\)

**Exact values of non-split two out degree equitable dominating for some special graphs**

**Peterson graph:** The non-split two out degree equitable domination number of Peterson graph is 5

**Proof:**

![Figure 2](image)

In the figure D= \(\{v_1, v_2\}\) is non split two out degree equitable dominating set

**Fan graph:** For any fan graph of order \(p \geq 4, \gamma_{ns2oe}(F_1,p-1) = \left\{\begin{array}{ll} 2 & \text{if } p = 4,5 \\ p - 2 & \text{if } p \geq 6 \end{array}\right.\)

![Figure 3](image)

In figure 4(a) D= \(\{v_1, v_2\}\) is non split two out degree equitable dominating set so \(\gamma_{ns2oe}(F_1,4) = 2\)
In figure 4(b) D= \(\{v_1, v_2, v_3, v_4\}\) is non-split two out degree equitable dominating set so \(\gamma_{ns2oe}(F_1,5) = 4\)

**Moser spindle:** The Moser spindle is an undirected graph with seven vertices and eleven edges as show in figure
In above figure $D=\{v_1,v_2,v_3\}$ is a non-split two out degree equitable dominating set so $\gamma_{nstoe}(G)=3$

**Bull Graph:** The Bull graph is a planner undirected graph with 5 vertices and 5 edges in the form of a triangle with two disjoint pendent edges

**Crown graph:** Any cycle with a pendent edge attached at each vertex is shown in figure 2 is called Crown graph and is denoted by $\mathcal{C}^+_p$.

For the Crown graph, $\gamma_{nstoe}(\mathcal{C}^+_p)=p$.

For figure 6 above theorem $D=\{v_1,v_2,v_3\}$ is minimum non split two out degree equitable dominating set.

2. METHODS & MATERIALS

Here we collect some research papers from various journals and downloads some papers from the internet related to the domination in graph theory and we defined a new domination number and we study them in details.

3. RESULTS

In this paper a new domination number called non split two out equitable domination number is defined and this domination number for some standard and special graphs called Peterson graph, diamond graph, Fan graph, Moser spindle and Bull graph.

4. CONCLUSIONS

In this paper a new domination number called non split two out equitable domination number is defined and this domination number for some standard and special graphs. Further we are like to expend this research work for another group of graphs and we like to study the applications of the non-split two out degree equitable domination number.

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