Tyre contact forces and calculation of translatory velocities.

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ABSTRACT

The simulation of a vehicle model involves the determination of horizontal forces at the tyre/road interface. The main task of tyre modeling is to derive these forces. The parameters that are essential for the estimation of horizontal forces are tyre slip, tyre side slip angle, and friction coefficients. These are the inputs to the force equation.

KEY WORDS: Translator, Velocities, Horizontal.

1. INTRODUCTION

Co-Efficient of Friction Calculation: Friction co-efficient is given by

\[ \mu = \frac{F_{\text{fric}}}{F_z} \]  

\( F_{\text{fric}} \) = Frictional force
\( F_z \) = Normal force

The frictional force is to be found out in the longitudinal direction as well as the lateral direction of the tyre. To find these forces the friction co-efficient are needed in those directions. As per Burckhardt’s approach the resultant friction co-efficient can be expressed in the direction of resultant slip and is given by the expression,

\[ \mu = (c_1(1 - e^{-c_2 s_{\text{rel}}}) - c_3 s_{\text{rel}}) \]  

Here \( c_1, c_2, c_3 \) are parameters for various road types. The friction co-efficient is influenced by high wheel velocities and heavy normal loads acting on the tyres. These effects should also be included in the friction co-efficient calculation.

Tyre Modeling Flowchart: Figure 1 shows the procedure and different parameters involved in tyre modeling in the form of flow chart.

Figure 1. Tyre modeling procedure

Hence two more parameters \( c_4, c_5 \) are included in the equation 3.2 Equation 3.2 now becomes

\[ \mu_{\text{relij}} = (c_1(1 - e^{-c_2 s_{\text{relij}}}) - c_3 s_{\text{relij}}) \cdot e^{-c_4 s_{\text{relij}} v_{\text{avg}}} \cdot (1 - c_5 F_{\text{ij}}^{-2}) \]  

Table 1. Values of \( c_1, c_2, c_3 \) for different road types

<table>
<thead>
<tr>
<th>Road Type</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asphalt dry</td>
<td>1.2801</td>
<td>23.99</td>
<td>0.52</td>
</tr>
<tr>
<td>Asphalt wet</td>
<td>0.857</td>
<td>33.822</td>
<td>0.347</td>
</tr>
<tr>
<td>Concrete</td>
<td>1.1973</td>
<td>25.168</td>
<td>0.5373</td>
</tr>
<tr>
<td>Cobblestones dry</td>
<td>1.3713</td>
<td>6.4565</td>
<td>0.6691</td>
</tr>
<tr>
<td>Cobblestones wet</td>
<td>0.4004</td>
<td>33.7080</td>
<td>0.1204</td>
</tr>
<tr>
<td>Snow</td>
<td>0.1946</td>
<td>94.129</td>
<td>0.0646</td>
</tr>
<tr>
<td>Ice</td>
<td>0.05</td>
<td>306.39</td>
<td>0</td>
</tr>
</tbody>
</table>

The values for parameters \( c_4, c_5 \) are given below:
The Equation 3.3 is given as a generalized format for all the wheels. The subscript “ij” represents front/rear, left/right tyres. Thus the friction coefficient for all the four tires are given by “equation (3.4)"

\[
\mu_{\text{resij}} = (c_1(1 - e^{-c_2 s_{\text{resij}}}) - c_3 s_{\text{resij}}) \cdot e^{-c_4 s_{\text{resij}} F_z} \cdot (1 - c_5 F_z^2)
\]

\[
\mu_{\text{refr}} = (c_1(1 - e^{-c_2 s_{\text{refr}}}) - c_3 s_{\text{refr}}) \cdot e^{-c_4 s_{\text{refr}} V_c} \cdot (1 - c_5 F_z^2)
\]

\[
\mu_{\text{resf}} = (c_1(1 - e^{-c_2 s_{\text{resf}}}) - c_3 s_{\text{resf}}) \cdot e^{-c_4 s_{\text{resf}} V_c} \cdot (1 - c_5 F_z^2)
\]

\[
\mu_{\text{resr}} = (c_1(1 - e^{-c_2 s_{\text{resr}}}) - c_3 s_{\text{resr}}) \cdot e^{-c_4 s_{\text{resr}} V_c} \cdot (1 - c_5 F_z^2)
\]

The friction coefficients in the longitudinal (\(\mu_l\)) and lateral (\(\mu_s\)) directions can be calculated as follows:

\[
\mu_{\text{lij}} = \mu_{\text{resij}} \frac{s_{\text{lij}}}{s_{\text{resij}}}
\]

\[
\mu_{\text{si}} = \mu_{\text{resij}} \frac{s_{\text{si}}}{s_{\text{resij}}}
\]

\(s_{\text{lij}}\) - Longitudinal slip
\(s_{\text{si}}\) - Lateral slip

The friction co-efficient in the lateral direction is less than that of in the longitudinal direction due to the tread profile. Hence an attenuation factor is used to correct the reduction. Usually the attenuation factor (\(k_s\)) lies in between 0.90 and 0.95. Thus \(\mu_s\) becomes

\[
\mu_{\text{si}} = k_s \cdot \mu_{\text{resij}} \frac{s_{\text{si}}}{s_{\text{resij}}}
\]

From the above equations it is evident to determine the slip values to find the coefficient of friction values.

**Slip Calculation:** Wheel slip occurs due to the difference of the rotational equivalent of tyre velocity (\(V_r\)) and the translatory velocity (\(V_w\)) of the wheel. The acceleration and braking forces are produced due to the presence of slip. There are two types of slip,

- Longitudinal slip
- Lateral slip

If the tyre rotates without lateral slip, tyre slip is the difference between the rotational equivalent velocity and translatory velocity. But the longitudinal slip and lateral slip occur simultaneously. Usually Longitudinal slip is considered in the direction of motion of the wheel (i.e.) in the direction of wheel velocity and the lateral slip in the direction perpendicular to the direction of wheel velocity. The rotational equivalent wheel velocity is in the direction of X axis of the wheel coordinate system whereas the wheel velocity is inclined at tyre side slip angle to it. It is evident from the following figure. Hence \(V_r\) should be multiplied by the cosine of tyre side slip angle before calculating slip. In slip calculations the speed difference is divided the respective larger speed to avoid division by zero.
Figure 2. Wheel slip calculation

The slip is calculated for driving and braking conditions for all the four wheels.

**Braking**

\[
\text{Longitudinal slip } s_{ij} = \frac{v_{ij} \cos \alpha_{ij} - v_{wij}}{v_{wij}}
\]  
(7)

\[
\text{Lateral slip } s_{ij} = \frac{v_{ij} \sin \alpha_{ij}}{v_{wij}}
\]

**Driving**

\[
\text{Longitudinal slip } s_{ij} = \frac{v_{ij} \cos \alpha_{ij} - v_{wij}}{v_{wij} \cos \alpha_{ij}}
\]  
(8)

\[
\text{Lateral slip } s_{ij} = \tan \alpha_{ij}
\]

Resultant slip is given by

\[
S_{resij} = \sqrt{S_{lij}^2 + S_{sij}^2}
\]  
(9)

- \(S_{resij}\) Resultant slip
- \(S_{lij}\) Longitudinal slip in the direction of \(v_{wij}\)
- \(S_{sij}\) Lateral slip in the direction perpendicular to \(v_{wij}\)
- \(\alpha_{ij}\) Tyre side slip angle
- \(v_{rij}\) Rotational equivalent wheel velocity of tyre
- \(v_{wij}\) Translatory wheel velocity

The subscript \(ij\) represents the front/rear, left/right tyres.

The three variables Tyre side slip angle, Rotational equivalent velocity of tyre and the translatory velocity of the tyre are calculated hereafter.

**Calculation of Rotational Equivalent Velocity of Tyre (\(v_{rij}\))**: The rotational equivalent velocity of the tyre is given by the expression

\[
v_{rij} = r_{effij} \cdot \omega_{ij}
\]  
(10)

- \(v_{rij}\) Rotational equivalent velocity of tyre
- \(r_{effij}\) Effective tyre radius
- \(\omega_{ij}\) Angular velocity of the tyre
- \(ij\) represents front/rear, left/right tyres

\(r_{eff}\) calculation

Let the tyre run over a length “\(l\)” for a time “\(t\)” through an angle “\(\phi\)”, then

\[
v_r = \frac{l}{t}, \quad \omega = \frac{\phi}{t}
\]

\[
= \frac{r_{eff}}{t} \cdot \frac{\phi}{t}
\]  
(11)

**Hence** \(r_{eff} = \frac{l}{\phi}\)

Consider Figure 3. Where the static and dynamic wheel radius is shown.
Calculation of Tyre Translatory Velocity: The tyre translatory velocity or the tyre road contact point velocity can be found by transforming the vehicle COG velocity to the tyre ground contact point. To perform such transformation the following variables are necessary,

- Vehicle COG velocity
- Yaw rate
- Distance between Vehicle COG and tyre road contact point
- Angle between the line joining Vehicle COG and tyre road contact point
- Tyre side slip angle
- Wheel turn angle

The vehicle COG velocity can be obtained from sensor data. The yaw rate is calculated as function of time \( t \). The other variables are calculated hereafter. The wheel turn angle can also be measured physically.

Distances between Vehicle Cog and Tyre Road Contact Points: The point of action of tyre force is the tyre ground contact point. It does not lie at the centre of the tyre road contact patch but it lies towards the rear due to wheel caster. It is evident from the Figure 4. It shows an aerial view of the tyre contact area including the tyre road contact point. The middle point of the tyre contact area migrates outwards and creates a torque with the longitudinal force which during acceleration increases self-aligning torque and during braking decreases it.

\[ r_{stat} = r_0 \cos \phi \quad ; \quad l = r_0 \sin \phi \]
\[ \phi = \cos^{-1} \left( \frac{r_{stat}}{r_0} \right) \]
\[ r_{eff} = r_0 \cdot \frac{\sin \left( \cos^{-1} \left( \frac{r_{stat}}{r_0} \right) \right)}{\cos^{-1} \left( \frac{r_{stat}}{r_0} \right)} \]
The caster is generally assumed to be constant and only the component in the direction of the wheel plane is considered. But, using the above method the caster can be calculated dynamically (with direction information). The frictional forces $F_l$ and $F_s$ act in the direction of the wheel velocity $v_w$ and in a direction perpendicular to it. The $F_{wl}$ and $F_{ws}$ can be obtained by transforming the forces $F_l$ and $F_s$ into the wheel coordinate system.

The distance between the tyre ground contact point and Vehicle COG ($rij$) can be calculated geometrically for each tyre. The angle between the line joining the Vehicle COG and the tyre/road contact point ($\gamma ij$) can also be found geometrically.

**Front Right Tyre**

\[ n_{ij} = \frac{1}{2} (-0.03 + 0.12 \frac{F_{lij}}{5000}) \]

\[ n_{sij} = 3n_{ij} \tan \alpha_{ij} \]

$m_{ij}$ Longitudinal caster of tyre

$l_{ij}$ Lateral caster of tyre

$F_{sij}$ Wheel vertical load

$\alpha_{ij}$ Tyre side slip angle

$i j$ represents front/rear, left/right tyres.

Front Left Tyre

\[ r_{fr} = \left( \frac{b}{2} + n_{x} \cos \delta_{x} + n_{y} \sin \delta_{x} \right)^2 \]

\[ \gamma_{fr} = \tan \left( \frac{\frac{b}{2} + n_{x} \cos \delta_{x} + n_{y} \sin \delta_{x}}{\frac{b}{2} + n_{x} \cos \delta_{x} + n_{y} \sin \delta_{x}} \right) \]
\[ r_f = \sqrt{(l_f + n_{id})^2 + (l_f/2 - n_{id})^2} \]
\[ \gamma_f = \tan^{-1} \left( \frac{l_f/2 - n_{id}}{n_{id}} \right) \]

**Distance between vehicle COG and front right tyre road contact point**

**Distance between vehicle COG and front left tyre road contact point**

**Distance between vehicle COG and rear left tyre road contact point**

**Distance between vehicle COG and rear right tyre road contact point**

**Angle between line joining vehicle COG, front right tyre road contact point and Vehicle COG coordinates system**

**Angle between line joining vehicle COG, front left tyre road contact point and Vehicle COG coordinates system**

**Angle between line joining vehicle COG, rear left tyre road contact point and Vehicle COG coordinates system**

**Angle between line joining vehicle COG, rear right tyre road contact point and Vehicle COG coordinates system**

Figure 7. Geometric calculation of the distance \( r_{rr} \)

Figure 8. Geometric calculation of the distance \( r_{rl} \)
2. RESULT AND DISCUSSION

In an automotive vehicle system safety and riding comfort are the primary important parameters and the roll over stability is an important parameter related with stability. Higher operating speeds and ride comfort are of greater importance for passenger vehicles, greater axle loads are mainly considered for cargo carriers. Hence it is essential to obtain a better understanding of the automotive system to run the vehicle at higher operating speeds with more safety. It is hoped that the models and the techniques presented in this thesis, would provide an answer to the problem of safety at high operations related to an automotive vehicle system.

3. CONCLUSION

Tyre construction: Thicker treads and increased number of carcass plies result in more work to be done by the tyre while rolling thus resulting in high hysterisis losses and high rolling resistance.

Road surface: Rolling resistance is less for hard, smooth and dry road surfaces. Rolling resistance is high for worn out roads since more work is done by the tyre to roll over the uneven surfaces. Rolling resistance is high for wet surfaces.

Tyre material: Synthetic rubber is having more rolling resistance than natural rubber. Butyl rubbers have more rolling resistance than synthetic rubber but have good road holding and traction properties.

Inflation pressure: Depending on the road type inflation pressure affects the rolling resistance. On hard surfaces when inflation pressure increases rolling resistance decreases. On deformable road surfaces more inflation pressure results in high ground penetration and more work is to be done by the tyre. If the inflation pressure is less, more deflection occurs resulting in high rolling resistance. So an optimum tyre pressure is to be maintained.

Speed: As speed of the rolling tyre increases the work done in deforming the tyre also increases and hence the rolling resistance increases. The multitude of factors discussed previously lead to complexity in the determination of rolling resistance. However in performance calculations it is sufficient to express co-efficient of rolling resistance as a linear function of speed.

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